**REQUIREMENTS FOR DISCIPLINE EXAMS: MATHEMATICS**

**FOR DOCTORAL SCHOOL STUDENTS**

The enclosed lists of topics and literature are indicative only. The examination commission (examiners), in consultation with the supervisor, determine the detailed scope of the discipline exam and the literature relevant for this exam, of which the doctoral student is informed.

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| **TOPICS:** |
| 1. Problem solving strategies, examples. 2. Functional equations and inequalities, what next? 3. Well-known Hilbert’s problems. 4. Graphs and their applications in various fields of mathematics. 5. Dirac’s Conjecture, Weak Dirac’s Conjecture, Green-Tao Theorem, A short proof of the Weak Dirac’s Conjecture for point configurations in . 6. The Collatz-Ulam problem in terms of stochastic methods and dynamical systems. 7. The Riemann Hypothesis. 8. The Fermat Last Theorem. 9. Selected similarities and differences between the notions of measure and category. 10. Characterization of extreme problems in the theory of point and line configurations. 11. Hirzebruch-type inequalities and their consequences in extreme problems in combinatorics. 12. Selected models of non-euclidean geometries. 13. The golden ratio in geometry. 14. How to arrange and evaluate projects? 15. Inductive and deductive methods in teaching mathematics. 16. How to do math? What is the job of a mathematician? 17. Variety of mathematical experience. 18. Why is mathematics useful? 19. Pure versus applied mathematics. 20. Philosophical difficulties of an active mathematician. 21. Nonstandard analysis. 22. Experiment versus proof in mathematics - methodology and examples. 23. Classical primality tests of natural numbers based on congruences. 24. Fermat and Mersenne numbers, prime factorizations and public-key cryptography. 25. Jordan measure of a set: definition, family of measurable sets, uniqueness, invariance, measure of an elementary figure, measure of similar figures, relations between the Jordan measure and the Riemann integral. 26. Geometry models over non-Archimedean fields. 27. Hilbert's axioms versus Euclid's axioms. 28. Axioms of the set theory, including the role of the axiom of choice. 29. Basic concepts of the category theory. 30. Basic problems in model theory, including consistency, completeness and decidability. 31. Selected topics of real analysis and functional analysis. 32. Smooth curves and smooth surfaces. 33. Local geometry of smooth surfaces. 34. Discrete stochastic processes. 35. Continuous stochastic processes. 36. Psychological and mathematical thinking processes. 37. Selected aspects of the process of learning and teaching mathematics. 38. The role of exemplary facts and activities specific to mathematics. 39. Different methods used to localize the spectrum of matrices. 40. Mathematical models – introduction, methods, and selected topics. |
| **LITERATURE:** Note that for some of the Polish titles below their original English versions are also available. |
| 1. M. Aigner, G. M. Ziegler, Dowody z Księgi, PWN, Warszawa 2002. 2. M. Alder, An Introduction to Mathematical Modelling, HeavenForBooks.com, 2001. 3. J. Borwein, D. Bailey, Mathematics by experiment: Plausible Reasoning in the 21st Century, AK Peters, Natick 2004. 4. J. Carlson, A. Jaffe, A. Wiles (Eds.), The millennium prize problems, AMS, Providence, Rhode Island, Clay Mathematics Institute, Cambridge, Massachusetts, 2006. 5. M. P. do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, 1976. 6. F. Corbalan, Złota proporcja. Matematyczny język piękna, RBA Colleccionables, Warszawa 2012. 7. P. J. Davis, R. Hersh, E. A. Marchisotto, The mathematical experience. With an introduction by Gian-Carlo Rota. Study edition. Birkhäuser Boston, Inc., Boston, MA, 1995. 8. P. Erdös, G. B. Purdy, Extremal problems in combinatorial geometry, w: R.L. Graham (Ed.) et al., Handbook of combinatorics. Vol. 1-2, Elsevier (North-Holland), Amsterdam, 1995, 809 — 874. 9. J. Gomez, Tam, gdzie proste są krzywe. Geometrie nieeuklidesowe, RBA Colleccionables, Warszawa 2012. 10. B. Grell, Wstęp do matematyki. Zbiory, struktury, modele, Wydawnictwo Uniwersytetu Jagiellońskiego, Kraków 2006. 11. G. R. Grimmett, D. R. Stirzaker, Probability and Random Processes, Oxford University Press, Oxford 2001. 12. G. Hardy, Apologia matematyka, Wyd. Prószyński i S-ka, Warszawa 1997. 13. M. Harris, Mathematics without apologies: Portrait of a problematic vocation, Princeton University Press, Princeton 2015. 14. R. Hartshorne, Geometry: Euclid and Beyond, Springer, New York 2000. 15. V. Klee, S. Wagon, Old and New Unsolved Problems in Plane Geometry and Number Theory, The Dolciani Mathematical Expositions 11, The Mathematical Association of America, Washington DC 1991. 16. S. G. Krantz, An Episodic History of Mathematics: Mathematical Culture Through Problem Solving, Mathematical Association of America, 2010. 17. S. G. Krantz, How to teach mathematics, AMS, Providence, Rhode Island, 2015. 18. S. Mac Lane, Mathematics Form and Function, Springer, New York 1986. 19. M. Marcus, H. Minc, A survey of matrix theory and matrix inequalities, Allyn and Bacon, Boston 1964. 20. Z. Moszner, O mierzeniu w matematyce, Biblioteczka matematyczna (tom 10), Państwowe Zakłady Wydawnictw Szkolnych, Warszawa 1961. 21. J. C. Oxtoby, Measure and Category, Graduate Texts in Mathematics, Springer-Verlag, New York Heidelberg Berlin 1971. 22. P. Ribenboim, Mała księga wielkich liczb pierwszych, WNT, Warszawa 1997. 23. T. Tao, Solving Mathematical Problems: A personal Perspective, Oxford University Press, Oxford 2006. 24. T. Tao, An Epsilon of Room, I: Real Analysis, Graduate Studies in Mathematics vol. 117, American Mathematical Society, Providence, Rhode Island, 2010. 25. D. Tall (Ed.), Advanced mathematical thinking, Mathematics Education Library vol. 11, Kluwer Academic Publishers, New York Boston Dordrecht London Moscow 1991. |